Ph 4207, Problem Set #4 (Total Marks 20)
(due 01/04/18)

1. Consider the 4 particle state,

$$|W\rangle = \frac{1}{2\sqrt{2}}(|1100\rangle + \sqrt{2}|1010\rangle - |1001\rangle + |0011\rangle - \sqrt{2}|0101\rangle + |0110\rangle). \tag{1}$$

Using the state given in Eq. 1, perform the task of teleporting $|\psi_2\rangle = \alpha_{00}|00\rangle + \alpha_{10}|10\rangle + \alpha_{01}|01\rangle + \alpha_{11}|11\rangle$, where, $|\alpha_{00}|^2 + |\alpha_{10}|^2 + |\alpha_{01}|^2 + |\alpha_{11}|^2 = 1$.
For teleporting this state, how many ebits of entanglement is needed between the parties. Give the detailed teleportation protocol, somewhat similar to the one done in the class, where a single qubit state was teleported using a two qubit entangled channel.
Here Alice is in possession of qubits 1 and 3, while Bob is in possession of qubits 2 and 4. [7]

2. Perform the dense coding protocol for the state $|W\rangle$, where again Alice possesses qubits 1 and 3, while Bob is left with 2 and 4. To perform the dense coding, Alice applies a set of unitary operators from $(I, \sigma_1, i\sigma_2, \sigma_3)$ to encode her classical information and sends her qubits to Bob as,

$$\sigma_i^1 \otimes \sigma_j^3 |W\rangle \rightarrow |W\rangle'. \tag{2}$$

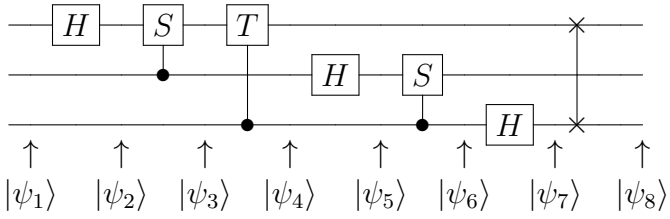
The operations of Alice are shown in table 1 in the left hand column. Fill in the the subsequent state obtained by Bob in the right hand column of this table.

Table 1: The operations of Alice and the subsequent state obtained by her.

| **operations by Alice** | **state obtained** |
|---|---|
| $I \otimes I$ | |
| $I \otimes \sigma_3$ | |
| $\sigma_3 \otimes I$ | |
| $\sigma_3 \otimes \sigma_3$ | |
| $I \otimes \sigma_1$ | |
| $I \otimes i\sigma_2$ | |
| $\sigma_3 \otimes \sigma_1$ | |
| $\sigma_3 \otimes i\sigma_2$ | |
| $\sigma_1 \otimes I$ | |
| $\sigma_1 \otimes \sigma_3$ | |
| $i\sigma_2 \otimes I$ | |
| $i\sigma_2 \otimes \sigma_3$ | |
| $\sigma_1 \otimes \sigma_1$ | |
| $i\sigma_1 \otimes \sigma_1$ | |
| $\sigma_1 \otimes i\sigma_2$ | |
| $i\sigma_2 \otimes \sigma_1$ | |
| $\sigma_2 \otimes \sigma_2$ | |

Bob then performs a joint von Neumann type measurement to obtain the classical information encoded by Alice. Show explicitly the states involved here by constructing a table. Since, these states are orthogonal to each other, they can be distinguished perfectly. [8]

3. Consider the following circuit




Consider
a) $|\psi_1\rangle = |000\rangle$
b) $|\psi_2\rangle = |001\rangle$
Find $|\psi_8\rangle$ in both cases (a) and (b). Evaluate all intermediate steps ($|\psi_2\rangle$,$|\psi_3\rangle$,...,$|\psi_7\rangle$) [5]